BOSWELL-BÈTA

James Boswell Exam
VWO Mathematics C

Date:	Practice exam 2
Time:	3 hours
Number of questions:	6
Number of subquestions:	23
Number of supplements:	0
Total score:	67

- Write your name **on every sheet** of paper you hand in.
- Use a separate sheet of paper for each question.
- For each question, show how you obtained your answer either by means of a calculation or, if you used a graphing calculator, an explanation. Otherwise, no points will be awarded to your answer.
- Make sure that your handwriting is legible and write in blue or black non-erasable ink. No correction fluid of any kind is permitted. Use a pencil only to draw graphs and geometric figures.
- You may use the following:
  - Graphing calculator;
  - Drawing utensils;
  - List of formulas;
  - Dictionary, subject to the approval of the invigilator.

Question 1: BMI and BSI

The BMI (Body Mass Index) is often used to determine whether a person is underweight or overweight. The formula for the BMI is:

BMI	meaning
lower than 18.5	underweight
from 18.5 to 25	healthy weight
from 25 to 30	overweight
30 or higher	obese

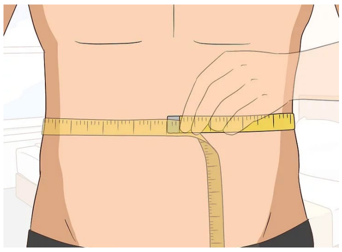
$$B = \frac{m}{l^2}$$

Here B is the BMI, m is the weight (in kg) and l is the length (in metres).

Danielle is 170 cm tall and has a healthy weight, see the table above.

4p **a** Calculate the smallest and the largest weight that Danielle can have. Give your answers rounded to whole kilograms.

Another measure that is used to determine whether someone is under- or overweight is the BSI (Body Shape Index).



The formula for the BSI is:

$$S = w \cdot m^{-\frac{2}{3}} \cdot l^{\frac{5}{6}}$$

Here S is the BSI, w is the waist circumference (in metres), m is the weight (in kg) and l is the length (in metres).

The value of your BSI gives information about your health risk. The table shows the guidelines for 20-year-old men.

BSI	health risk
lower than 0.074	very low
from 0.074 to 0.076	low
from 0.076 to 0.078	average
from 0.078 to 0.080	high
0.080 or higher	very high

Eric is 20 years old. He is 1.80 metres tall and weighs 87.5 kg. His waist circumference is 96 cm.

2p **b** Calculate the health risk of Eric (according to the table).

3p **c** Write the formula $S = w \cdot m^{-\frac{2}{3}} \cdot l^{\frac{5}{6}}$ without negative or fractional exponents.

Question 2: Braille

Braille is an alphabet developed especially for the blind. The characters are formed by dots in a 3×2 grid. Each dot is either tactile or not tactile. Below are the first five letters of the Braille alphabet. A black dot ($\bullet$) is tactile, an open circle ($\circ$) is not.

●	○	●	○	●	●	●	●	●	○
○	○	●	○	○	○	○	●	○	●
○	○	○	○	○	○	○	○	○	○
a		b		c		d		e	

3p **a** How many characters are possible in which exactly one or two dots are tactile?

A Braille display is an aid for blind people when using computers, tablets and smartphones.

On a Braille display, the characters are formed by eight dots instead of six. This increases the total number of characters you can make with the dots.

3p **b** How many extra characters are there when using eight dots (instead of six dots)? [1)]
Exclude the 'character' with no tactile dots in this calculation.

In 2015, approximately 36 million people worldwide were blind.
The world population in 2015 was approximately equal to 7.380 billion.

2p **c** Calculate what percentage of the world's population was blind in 2015. Round your answer to two decimal places.

Researchers have predicted that the number of blind people worldwide will triple in the period $2015 - 2050$. The main reasons for this increase are population growth and increased life expectancy.

Assume that this prediction is correct and that the number of blind people increases exponentially in the period $2015 - 2050$.

3p **d** Calculate by what percentage **per year** the number of blind people will increase during the period $2015 - 2050$. Round your answer to one decimal place.

1) This question is about *all* possible characters (not just the ones with one or two tactile dots)

Question 3: Test

Amir and Bo are high school students. They both have to take a test soon. They are having a discussion about what it takes to pass. We can summarise this discussion using the following abbreviations:

C: You chatter in class.

Q: You ask questions.

E: You do all the exercises.

P: You pass the test.

Amir claims the following:

$$\neg C \wedge Q \wedge E \Rightarrow P$$

Bo doesn't quite agree with Amir. Bo claims:

$$(\neg C \vee Q) \wedge E \Rightarrow P$$

2p **a** Explain the difference of opinion between Amir and Bo.

Another student agrees with Amir and concludes: 'If you don't pass the test, then you chatter in class or you don't do all the exercises or you don't ask questions'.

3p **b** Write this conclusion using the abbreviations above and logic symbols.

The test consists of ten multiple choice questions. Each question has three possible answers: A, B or C. Only one of these is correct.

We make the following assumptions for students who are well prepared for the test:

- The probability that such a student chooses the correct answer to a question, is 0.8.
- The probability that such a student chooses one incorrect answer is 0.1.
- The probability that such a student chooses the other incorrect answer is also 0.1.

Two students who are both well prepared take the test.

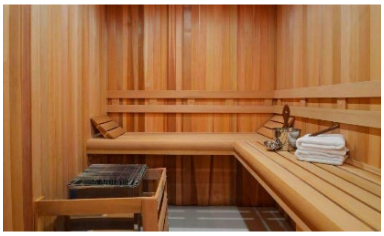
3p **c** The probability of these students giving the same answer to a random question is 0.66. Show this with a calculation.

When grading the test, the teacher notices that the two students have given exactly the same answer to each question. So they got the same questions right and in the other questions they chose the same wrong answer.

The teacher wonders if the students cheated.
He wants to know the probability that
two well-prepared students, who do not cheat, give
the same answer to all ten questions.

2p **d** Calculate this probability accurate to three decimal places.

Question 4: Sauna



At 12:00 the heating element of a sauna is turned on. This causes the temperature in the sauna to rise.

Between 12:00 and 12:45, the temperature in the sauna increases from 20 °C to 56.3 °C.

Suppose that the temperature in the sauna increases linearly.

3p **a** Use linear interpolation to calculate the temperature in the sauna at 12:10.
Give your answer in °C and round to one decimal place.

In reality, the temperature does not increase linearly, but according to the formula:

$$S = 200 - 180 \cdot 0{,}741^{t}$$

Here S is the temperature (in °C) and t is the time (in hours) where $t = 0$ corresponds to 12:00.

4p **b** Calculate the percentage increase in temperature between 13:00 and 13:15.
Round your answer to one decimal place.

To determine how long it takes for the sauna to reach a certain temperature, it is convenient to rewrite the formula

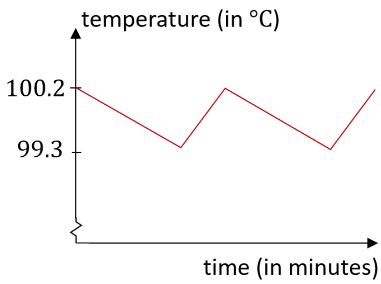
$$S = 200 - 180 \cdot 0{,}741^{t}$$

into a form such that t is expressed in terms of S.

5p **c** Express t in terms of S **and** use this rewritten formula to calculate the time at which the sauna reaches 100 °C. Write your answer in the form ‘hh:mm’.

When the temperature reaches 100 °C, the heating process stops and the temperature is kept constant.

In reality, the temperature then still fluctuates between 100.2 °C and 99.3 °C. See the graph to the right.



This graph is periodic.

2p **d** Calculate the equilibrium (mean value) and the amplitude of the graph.

Question 5: Forest

In a forest, a fixed proportion of the trees are cut down every year. In return, every year new trees are planted.

The number of trees T (in the forest) after n years is described by the recursive formula:

$$T_n = 0{,}9 \cdot T_{n-1} + 2500 \text{ with } T_0 = 10\,000$$

2p **a** Explain the practical meaning of the numbers 0.9 and 2500 in this formula.

The number of trees in the forest increases.

3p **b** Calculate the increase in the number of trees during the first three years.

Maria is an ecologist. She studies the different plant species that grow in the forest. To this end, she divides the forest into hundreds of rectangular pieces of land, see the figure.

She then takes a random sample of a number of these pieces of land and determines which plant species grow on them.

A certain plant species grows on half of all rectangular pieces of land in the forest.

3p **c** Suppose that Maria's sample consists of twenty pieces of land.
Calculate the probability that the plant species grows on at most seven (of the twenty) pieces of land. Round your answer to three decimal places.

Another plant species grows on 5% of all rectangular pieces of land in the forest.

4p **d** Determine the number of pieces of land that Maria's sample should consist of, so that the probability that she will find the plant species on at least one of these pieces of land is greater than 90%.

Question 6: Golf balls

Golf balls may be only be used in professional matches if they meet certain requirements.

One such requirement is that a golf ball must not weigh more than 1.62 ounces (1 ounce ≈ 28.35 grams).

A manufacturer of sports goods produces golf balls. The weight of the golf balls is normally distributed with a mean of 45.5 grams and a standard deviation of 0.15 gram.

3p **a** Calculate the percentage of golf balls that weigh more than is allowed. Round your answer to two decimal places.

A second requirement for golf balls is that their diameter must not be too small.

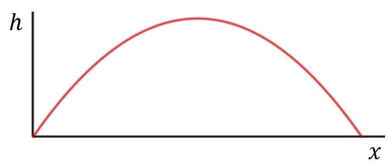
The diameter of the manufacturer's golf balls is normally distributed with a mean of 43.25 millimetres and a standard deviation of 0.25 millimetres. 1% of the golf balls has a smaller diameter than is allowed.

2p **b** Calculate the minimum diameter that a golf ball must have (in order to be allowed). Give your answer in millimetres and round to two decimal places.

A third requirement that golf balls have to meet, is that they should not travel too far.

A golf ball may only be used in professional matches if (in a certain test) the horizontal distance travelled by the ball does not exceed 320 yards (1 yard ≈ 0.9144 metre).

You see a graph of the trajectory of a golf ball. The formula of this graph is:



$$h = -\frac{1}{900}(x-150)^2 + 25$$

Here h is the height and x is the horizontal distance to the place where the golf ball is hit, both in metres.

3p **c** Determine whether the horizontal distance travelled by this golf ball exceeds 320 yards.

3p **d** You can write the formula $h = -\frac{1}{900}(x-150)^2 + 25$ in the form $h = ax^2 + bx$. Show this and write down the values of a and b.

END OF EXAM